The Solution Of The 1st PS

Epsilite Team

Part 1: Ideal Gas Laws

Q1: Final Pressure (P_2)

Given a constant temperature and mass, we use Boyle's Law:

$$P_1V_1 = P_2V_2$$

Given: $V_1 = 540$ cm^3, $P_1 = 63.3$ kPa, $V_2 = 325$ cm^3

$$P_2 = \frac{P_1V_1}{V_2} = \frac{63.3 \times 540}{325} \approx 105.25 \text{ kPa}$$

Q2: Pressure in a Rigid Container

For a rigid container, volume is constant. We use Gay-Lussac's Law:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Given: $P_1 = 1$ atm, $T_1 = 25°\text{C} = 298.15$ K, $T_2 = 75°\text{C} = 348.15$ K.

$$P_2 = \frac{P_1T_2}{T_1} = \frac{1 \times 348.15}{298.15} \approx 1.17 \text{ atm}$$

Q3: Final Temperature (T_2)

Using the Combined Gas Law:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Given: $V_1 = 10$ L, $T_1 = 24°\text{C} = 297.15$ K, $P_1 = 80$ kPa. $V_2 = 20$ L, $P_2 = 107$ kPa.

$$T_2 = \frac{P_2V_2T_1}{P_1V_1} = \frac{107 \times 20 \times 297.15}{80 \times 10} \approx 794.88 \text{ K}$$

Part 2: Calorimetry and Heat Capacity

Exercise 1 - Question 1

The calorimetric equation for the system (water + calorimeter) is:

$$Q_{cold} + Q_{cal} + Q_{hot} = 0$$

$$(m \cdot c_w + C)(\theta_{eq} - \theta_1) + m \cdot c_w(\theta_{eq} - \theta_2) = 0$$

Given: $m = 200$ g $= 0.2$ kg, $\theta_1 = 15.1$°C, $\theta_2 = 52.3$°C, $\theta_{eq} = 32.4$°C.

$$C = \frac{m \cdot c_w(\theta_2 - \theta_{eq})}{\theta_{eq} - \theta_1} - m \cdot c_w$$

$$C = \frac{0.2 \times 4185 \times (52.3 - 32.4)}{32.4 - 15.1} - (0.2 \times 4185) \approx 125.17 \text{ J K}^{-1}$$

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Exercise 1 - Question 2

Neglecting calorimeter heat capacity ($C = 0$):

$$m_w c_w(\theta_{eq} - \theta_0) + m_{block} c_{block}(\theta_{eq} - \theta_{block}) = 0$$

Given: $m_w = 0.2$ kg, $\theta_0 = 18$°C, $m_{block} = 0.3$ kg, $\theta_{block} = 90$°C.

Aluminum (Al): $\theta_{Al} = 35.5$°C.

$$c_{Al} = \frac{m_w c_w(\theta_{Al} - \theta_0)}{m_{block}(\theta_{block} - \theta_{Al})} = \frac{0.2 \times 4185 \times (35.5 - 18)}{0.3 \times (90 - 35.5)} \approx 896.22 \text{ J K}^{-1}\text{kg}^{-1}$$

$$C_{m,Al} = c_{Al} \times M_{Al} = 896.22 \times 0.02698 \approx 24.18 \text{ J mol}^{-1}\text{K}^{-1}$$

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Nickel (Ni): $\theta_{Ni} = 27.9$°C.

$$c_{Ni} = \frac{m_w c_w(\theta_{Ni} - \theta_0)}{m_{block}(\theta_{block} - \theta_{Ni})} = \frac{0.2 \times 4185 \times (27.9 - 18)}{0.3 \times (90 - 27.9)} \approx 444.89 \text{ J K}^{-1}\text{kg}^{-1}$$

$$C_{m,Ni} = c_{Ni} \times M_{Ni} = 444.89 \times 0.0587 \approx 26.11 \text{ J mol}^{-1}\text{K}^{-1}$$

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Part 3: Nitrogen Gas and Phase Change

Exercise A: Nitrogen Bottle

1. **Mass of N_2**: $PV = \frac{m}{M}RT \implies m = \frac{PVM}{RT}$. Using $P = 100$ bar $= 10^7$ Pa, $V = 1.8$ L $= 1.8 \times 10^{-3}$ m^3, $T = 283.15$ K, $M = 28$ g/mol.

$$m = \frac{10^7 \times 1.8 \times 10^{-3} \times 28}{8.314 \times 283.15} \approx 214.1 \text{ g}$$

2. **New Pressure at $T_2 = 38°$C (311.15 K)**:

$$P_2 = \frac{P_1 T_2}{T_1} = \frac{100 \times 311.15}{283.15} \approx 109.89 \text{ bar}$$

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3. **Mass to release to keep $P = 100$ bar at $38°$C**:

$$m_{final} = \frac{PVM}{RT_2} = \frac{10^7 \times 1.8 \times 10^{-3} \times 28}{8.314 \times 311.15} \approx 194.8 \text{ g}$$

$$\Delta m = 214.1 - 194.8 = 19.3 \text{ g}$$

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Exercise B: Ice Cube Transformation

Total heat $Q_{total} = Q_{ice} + Q_{fusion} + Q_{water}$.

$$Q_{total} = mc_{ice}\Delta T_1 + mL_f + mc_w\Delta T_2$$

Given: $m = 0.05$ kg, $T_{init} = -10°$C, $T_{final} = 20°$C.

$$Q_{total} = (0.05 \times 2100 \times 10) + (0.05 \times 334 \times 10^3) + (0.05 \times 4185 \times 20)$$

$$Q_{total} = 1050 + 16700 + 4185 = 21935 \text{ J}$$

Part 4: Calorimetry and Melting Equilibrium

Exercise 2

Water equivalent $\mu = 20$ g $\implies C_{cal} = 0.02 \times 4185 = 83.7$ J/K[cite: 37, 38]. Heat released by water and calorimeter cooling to 0°C:

$$Q_{avail} = (m_w c_w + C_{cal})(30 - 0) = (0.2 \times 4185 + 83.7) \times 30 = 27621 \text{ J}$$

1. **Mass of melted ice**:

$$m_{melted} = \frac{Q_{avail}}{L_f} = \frac{27621}{334000} \approx 0.0827 \text{ kg} = 82.7 \text{ g}$$

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2. **Remaining ice**:

$$m_{rem} = 100 \text{ g} - 82.7 \text{ g} = 17.3 \text{ g}$$

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