Solution of PS 02:

Epsilite team

Exercise 1: Tire Pressure and Temperature

1. Calculation of the new pressure P_2:
Using Gay-Lussac's Law ($\frac{P_1}{T_1} = \frac{P_2}{T_2}$):

$$\begin{aligned} T_1 &= 25 + 273.15 = 298.15 \text{ K} \\ T_2 &= 50 + 273.15 = 323.15 \text{ K} \\ P_2 &= P_1 \times \frac{T_2}{T_1} = 3 \times \frac{323.15}{298.15} \approx \mathbf{3.25} \text{ atm} \end{aligned}$$

2. Risk Assessment:
Since $P_2(3.25 \text{ atm}) < P_{max}(6 \text{ atm})$, there is **no risk** of bursting.

Exercise 2: Ideal Gas Cycle Analysis

Q1: Nature of Transformations

- **C → A:** Isobaric ($P = \text{const}$).
- **A → B:** Isothermal ($T = \text{const}$).
- **B → C:** Adiabatic ($Q = 0$).

Q2: Detailed Calculation of State Variables (P, V, T)

We use $R = 0.082 \text{ L} \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$ and $n = 1$ mol.

Step 1: State A (Initial)

From the graph: $P_A = 1$ atm, $T_A = 273$ K.

$$V_A = \frac{nRT_A}{P_A} = \frac{1 \times 0.082 \times 273}{1} = \mathbf{22.386} \text{ L}$$

Step 2: State B (Isothermal from A)

From the graph: $T_B = T_A = 273$ K and $P_B = 2$ atm.

$$V_B = \frac{nRT_B}{P_B} = \frac{1 \times 0.082 \times 273}{2} = \mathbf{11.193} \text{ L}$$

Step 3: State C (Adiabatic from B)

From the graph: $P_C = 1$ atm. For an adiabatic process, we use Laplace's Law:

$$P_B^{1-\gamma} T_B^{\gamma} = P_C^{1-\gamma} T_C^{\gamma} \implies T_C = T_B \left(\frac{P_B}{P_C}\right)^{\frac{1-\gamma}{\gamma}}$$

Given $\gamma = 1.4$, then $\frac{1-\gamma}{\gamma} = \frac{-0.4}{1.4} \approx -0.2857$.

$$T_C = 273 \times \left(\frac{2}{1}\right)^{-0.2857} = 273 \times (0.5)^{0.2857}$$
$$T_C \approx 273 \times 0.8203 = \mathbf{223.95} \text{ K}$$

Now, find V_C:

$$V_C = \frac{nRT_C}{P_C} = \frac{1 \times 0.082 \times 223.95}{1} = \mathbf{18.364} \text{ L}$$

Q3: Summary Table of Quantities (W, Q, $\Delta U, \Delta H$)

Using $C_v = 20.785$ J/mol $\cdot$ K and $C_p = 29.099$ J/mol $\cdot$ K.

Process	ΔU (J)	Q (J)	W (J)	ΔH (J)
$C \to A$	+1019.5	+1427.3	−407.8	+1427.3
$A \to B$	0	−1573.3	+1573.3	0
$B \to C$	−1019.5	0	−1019.5	−1427.3
Cycle	**0**	**-146.0**	**+146.0**	**0**

4. Conclusion

- **First Law:** $\Delta U_{cycle} = \sum \Delta U = 0$, and $W_{net} + Q_{net} = 0$, verifying the law.
- **Cycle Nature:** Since $W_{net} > 0$, the cycle is a **Reciever Cycle** (consumes work).

Exercise 3: Heat Engine Cycle

Q1: Calculation of the ratio γ

Given $C_v = 2.9$ cal/mol $\cdot$ K and $R = 2$ cal/mol $\cdot$ K.
Using Mayer's relation ($C_p = C_v + R$):

$$C_p = 2.9 + 2 = 4.9 \text{ cal/mol} \cdot \text{K}$$
$$\gamma = \frac{C_p}{C_v} = \frac{4.9}{2.9} \approx \mathbf{1.69}$$

Q2: Calculation of Q, W, and ΔU (using $R = 8.314$ J/mol·K)

Constants in Joules (1 cal = 4.184 J):
$C_v = 12.13$ J/mol · K, $C_p = 20.50$ J/mol · K.

1. **A → B (Adiabatic):** $Q_{AB} = 0$ J.
   $\Delta U_{AB} = W_{AB} = nC_v(T_B - T_A) = 1 \times 12.13 \times (470 - 350) = \mathbf{1455.6}$ J.

2. **B → C (Isobaric):** $T_C = T_B \frac{V_C}{V_B} = 470 \times \frac{5.0}{4.5} \approx 522.2$ K.
   $Q_{BC} = nC_p(T_C - T_B) = 1 \times 20.50 \times 52.2 = \mathbf{1070.1}$ J.
   $\Delta U_{BC} = nC_v(T_C - T_B) = 633.2$ J.
   $W_{BC} = \Delta U - Q = \mathbf{-436.9}$ J.

3. **C → D (Adiabatic):** $Q_{CD} = 0$ J. Assuming $T_D \approx 388.9$ K from cycle properties:
   $\Delta U_{CD} = W_{CD} = nC_v(T_D - T_C) = \mathbf{-1617}$ J.

4. **D → A (Isochoric):** $W_{DA} = 0$ J.
   $Q_{DA} = \Delta U_{DA} = nC_v(T_A - T_D) = \mathbf{-471.9}$ J.

Q3: Proof of Thermal Efficiency R_t

The efficiency is defined as $R_t = 1 - \frac{|Q_{out}|}{Q_{in}}$.

- Q_{in} occurs during $B \to C$ (Heating): $Q_{in} = nC_p(T_C - T_B)$.
- Q_{out} occurs during $D \to A$ (Cooling): $Q_{out} = nC_v(T_A - T_D)$.

Substituting these into the efficiency formula:

$$R_t = 1 - \frac{|nC_v(T_A - T_D)|}{nC_p(T_C - T_B)}$$
$$R_t = 1 - \frac{C_v(T_D - T_A)}{C_p(T_C - T_B)} \quad \text{Since } \frac{C_v}{C_p} = \frac{1}{\gamma}$$
$$R_t = \left| 1 - \frac{T_D - T_A}{\gamma(T_C - T_B)} \right|$$

$\square$

Exercise 5: Calorimetry

1. Heat Balance Equation

In an isolated system (the calorimeter), the sum of heat exchanged is zero:

$$\sum Q = Q_{cal} + Q_{water} + Q_{lead} = 0$$

Where:

- $Q_{cal} = C_{cal}(T_{eq} - T_1)$ (Heat gained by calorimeter)
- $Q_{water} = m_1 C_{water}(T_{eq} - T_1)$ (Heat gained by water)
- $Q_{lead} = m_2 C_{p,lead}(T_{eq} - T_2)$ (Heat lost by lead)

2. Calculation of $C_{p,lead}$

Substituting the expressions into the balance equation:

$$(C_{cal} + m_1 C_{water})(T_{eq} - T_1) + m_2 C_{p,lead}(T_{eq} - T_2) = 0$$

Solving for $C_{p,lead}$:

$$C_{p,lead} = \frac{-(C_{cal} + m_1 C_{water})(T_{eq} - T_1)}{m_2(T_{eq} - T_2)}$$

Numerical Application:

- $C_{cal} = 209$ J/K
- $m_1 C_{water} = 350 \text{ g} \times 4.185 \text{ J/(g} \cdot \text{K)} = 1464.75 \text{ J/K}$
- $\Delta T_{cold} = 17.7 - 16 = 1.7°\text{C}$
- $\Delta T_{lead} = 17.7 - 98 = -80.3°\text{C}$

$$\begin{aligned}
C_{p,lead} &= \frac{-(209 + 1464.75) \times 1.7}{280 \times (-80.3)} \\
C_{p,lead} &= \frac{-2845.375}{-22484} \\
C_{p,lead} &\approx \mathbf{0.1265} \text{ J/(g} \cdot \text{K)}
\end{aligned}$$

$\square$