Set 3 correction

Detailed Correction — Entropy, Temperature Scales, and Thermodynamic Cycles

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Part I: Entropy

Exercise 1.1: *Ice Cube Entropy*

Given data:

- Mass of ice: $m = 5\,\text{g}$
- Initial temperature of ice: $T_0 = 0\,°\text{C} = 273.15\,\text{K}$
- Temperature of the lake (thermostat): $T_f = 17\,°\text{C} = 290.15\,\text{K}$
- Specific heat of water: $c_{\text{water}} = 4.18\,\text{J/g·K}$
- Latent heat of fusion: $L_f = 334\,\text{J/g}$

1. Entropy variation of the ice cube (ΔS_{ice})
The process has two stages: melting at T_0, then heating from T_0 to T_f.

$$\begin{aligned}\Delta S_{\text{ice}} = \Delta S_{\text{melting}} + \Delta S_{\text{heating}} &= \frac{mL_f}{T_0} + mc_{\text{water}} \ln\left(\frac{T_f}{T_0}\right) \\ &= \frac{5 \times 334}{273.15} + 5 \times 4.18 \times \ln\left(\frac{290.15}{273.15}\right) \\ &= 6.114 + 20.9 \times 0.06038 \approx \boxed{7.376\,\text{J/K}}\end{aligned}$$

2. Entropy variation of the lake (ΔS_{lake})
The lake is a thermostat at constant T_f. It releases the heat absorbed by the ice:

$$Q_{\text{absorbed by ice}} = mL_f + mc_{\text{water}}(T_f - T_0) = (5 \times 334) + 5 \times 4.18 \times 17 = 1670 + 355.3 = 2025.3\,\text{J}$$

$$\Delta S_{\text{lake}} = \frac{-Q_{\text{absorbed}}}{T_f} = \frac{-2025.3}{290.15} \approx \boxed{-6.980\,\text{J/K}}$$

3. Entropy of the universe (created entropy)

$$\Delta S_{\text{univ}} = \Delta S_{\text{ice}} + \Delta S_{\text{lake}} = 7.376 - 6.980 \approx \boxed{+0.396\,\text{J/K}}$$

4. Reversibility
No, the transformation is **irreversible**. By the Second Law of Thermodynamics, $\Delta S_{\text{univ}} > 0$ strictly, whereas a reversible process requires $\Delta S_{\text{univ}} = 0$.

Part II: Temperature Scales

Exercise 2.1: *Linear Scale $\mathcal{N}$*

The linear relationship has the form $\mathcal{N} = aC + b$. Applying the two fixed points:

- Melting ice: $0\,°\text{C} \leftrightarrow 100\,°\mathcal{N} \;\Rightarrow\; b = 100$
- Boiling water: $100\,°\text{C} \leftrightarrow 400\,°\mathcal{N} \;\Rightarrow\; 100a + 100 = 400 \;\Rightarrow\; a = 3$

The empirical law is therefore $\mathcal{N} = 3C + 100$.

1. Value of $150\,°\text{C}$ in the $\mathcal{N}$ scale:

$$\mathcal{N} = 3(150) + 100 = \boxed{550\,°\mathcal{N}}$$

2. Temperature where both scales give the same numerical value — set $\mathcal{N} = C = x$:

$$x = 3x + 100 \;\Rightarrow\; -2x = 100 \;\Rightarrow\; \boxed{x = -50}$$

Both scales read -50 at $-50\,°\text{C} = -50\,°\mathcal{N}$.

Exercise 2.2: *Logarithmic Thermometer*

a) The main underlying phenomenon is the **thermal volume expansion** of the thermometric liquid: a temperature change causes a proportional change in volume, and hence in the column height x.

b) Determination of constants a and b in $t = a\ln(x) + b$:

Applying the two reference states:

$$0 = a\ln(5) + b \;\Rightarrow\; b = -a\ln(5)$$
$$100 = a\ln(25) + b = a\ln(25) - a\ln(5) = a\ln\left(\tfrac{25}{5}\right) = a\ln(5)$$

$$\boxed{a = \frac{100}{\ln 5} \approx 62.13\,°\text{C}} \qquad \boxed{b = -100\,°\text{C}}$$

The empirical law is $t = 62.13\ln(x) - 100$.

c) Height differences. Inverting the law gives $x(t) = 5 \cdot 5^{\,t/100}$.

- Between $0\,°\text{C}$ and $10\,°\text{C}$:

$$x(0) = 5\,\text{cm}, \qquad x(10) = 5 \cdot 5^{0.1} \approx 5.873\,\text{cm}$$
$$\Delta x_1 = \boxed{0.873\,\text{cm}}$$

- Between $90\,°\text{C}$ and $100\,°\text{C}$:

$$x(90) = 5 \cdot 5^{0.9} \approx 21.283\,\text{cm}, \qquad x(100) = 25\,\text{cm}$$
$$\Delta x_2 = \boxed{3.717\,\text{cm}}$$

The non-linearity is clear: a $10\,°C$ step produces a much larger height change at the high end than at the low end.

Part III: First and Second Laws — Thermodynamic Cycle

Exercise 3.1:

1. Direction and nature

a) Nature of each transformation:

- $A \to B$: **Isochoric heating** — vertical upward; V constant, P and T increase.
- $B \to C$: **Isobaric compression** — horizontal leftward; P constant, V and T decrease.
- $C \to D$: **Adiabatic compression** — curve up-left, $Q = 0$; V decreases, P and T increase.
- $D \to E$: **Isochoric cooling** — vertical downward; V constant, P and T decrease.
- $E \to A$: **Adiabatic expansion** — curve down-right, $Q = 0$; V increases, P and T decrease.

b) The cycle is traced **counter-clockwise** on the P-V diagram, which corresponds to a **receiver** (refrigerator / heat pump) cycle: the net work received by the gas is positive ($W_{\text{cycle}} > 0$).

2. Sign table

Convention used: $\Delta U = W + Q$; expansion $\Rightarrow W < 0$, compression $\Rightarrow W > 0$.

Process		ΔT	W	Q	ΔU	ΔH	ΔS
$A \to B$	Isochoric heating	+	0	+	+	+	+
$B \to C$	Isobaric compression	−	+	−	−	−	−
$C \to D$	Adiabatic compression	+	+	0	+	+	0
$D \to E$	Isochoric cooling	−	0	−	−	−	−
$E \to A$	Adiabatic expansion	−	−	0	−	−	0
Complete cycle		0	+	−	0	0	0

3. Expression of T_A as a function of T_B and T_C

Entropy is a state function, so $\Delta S_{\text{cycle}} = 0$:

$$\Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DE} + \Delta S_{EA} = 0$$

For each reversible path ($\Delta S_{CD} = \Delta S_{EA} = 0$):

$$nC_v \ln\left(\frac{T_B}{T_A}\right) + nC_p \ln\left(\frac{T_C}{T_B}\right) + nC_v \ln\left(\frac{T_E}{T_D}\right) = 0$$

Dividing by nC_v and using $C_p/C_v = \gamma$, then substituting $T_D = T_B$, $T_E = T_C$:

$$\ln\left(\frac{T_B}{T_A}\right) + \gamma \ln\left(\frac{T_C}{T_B}\right) + \ln\left(\frac{T_C}{T_B}\right) = 0$$

$$\ln\left(\frac{T_B}{T_A}\right) = -(\gamma+1)\ln\left(\frac{T_C}{T_B}\right) = (\gamma+1)\ln\left(\frac{T_B}{T_C}\right)$$

Taking the exponential:

$$\frac{T_B}{T_A} = \left(\frac{T_B}{T_C}\right)^{\gamma+1} \implies \boxed{T_A = T_C\left(\frac{T_C}{T_B}\right)^{\gamma}}$$

Numerical application — $\gamma = \frac{5}{3}$, $T_B = 835\,\text{K}$, $T_C = 555\,\text{K}$:

$$T_A = 555 \times \left(\frac{555}{835}\right)^{5/3} = 555 \times (0.6647)^{1.6667} = 555 \times 0.5057 \approx \boxed{280.7\,\text{K}}$$