Electrostatics

Complete Exercise Solutions

Physics — Charges, Forces & Fields

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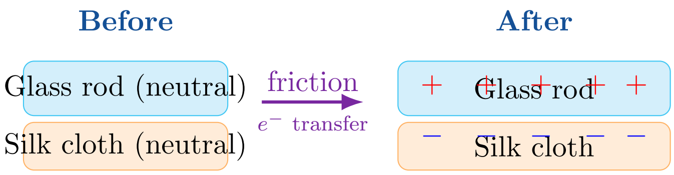
Given: Before rubbing, both objects are electrically neutral. During rubbing, $n = 30 \times 10^{10}$ electrons are transferred from glass to silk.

(a) Explanation — Triboelectric Effect

This phenomenon is called the **Triboelectric Effect** (charging by friction).

- Before rubbing, both objects are **electrically neutral** (equal protons and electrons).
- Friction causes electrons to transfer **from glass to silk**.
- Glass has a weaker hold on its electrons, so it loses them to silk.
- After the process: glass becomes **positively charged**, silk becomes **negatively charged**.
- **Total charge is conserved** — no charge is created or destroyed, only transferred.



(b) Charge on the Glass Rod

$$Q_{glass} = n \times e = 30 \times 10^{10} \times 1.6 \times 10^{-19} = 48 \times 10^{-9} \text{ C}$$

$$\boxed{Q_{glass} = +4.8 \times 10^{-8} \text{ C} = +48 \text{ nC}}$$

The glass rod is **positively charged** (lost electrons).

(c) Charge on the Silk Cloth

By conservation of charge:

$$\boxed{Q_{silk} = -4.8 \times 10^{-8} \text{ C} = -48 \text{ nC}}$$

The silk is **negatively charged** (gained electrons).

Object	Before	After	Sign
Glass Rod	0 C	$+4.8 \times 10^{-8}$ C	+
Silk Cloth	0 C	-4.8×10^{-8} C	−
Total	**0 C**	**0 C**	✓

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Given: $d = 1$ cm (ball diameter), $L = 10$ cm (string length), $\alpha = 10°$ (angle with vertical), $\rho_{Al} = 2700$ kg/m^3.

(a) Steps of the Electrophorus

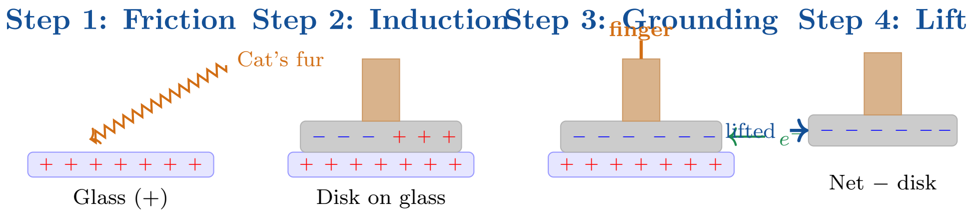

Figure 1: Four steps of the electrophorus charging process

1. **Charging by friction:** Glass becomes positively charged via the triboelectric effect.
2. **Induction:** Metal disk develops charge separation — negative on bottom, positive on top.
3. **Grounding:** Finger touch lets Earth electrons neutralize the positive top.
4. **Lifting:** Remove finger first, then disk — disk retains net **negative charge**. Glass plate is reusable.
5. **Contact with balls:** Charge distributes equally to both identical balls — both become negative and repel.

(b) Free Body Diagram

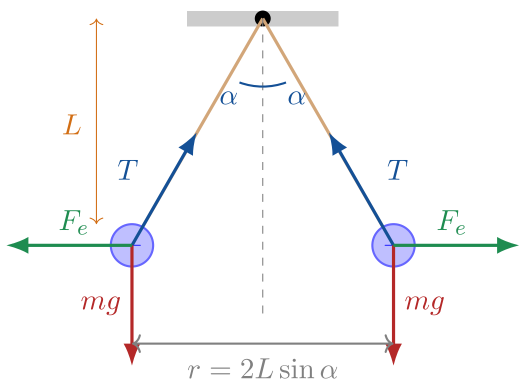

Figure 2: Free body diagram: tension T, weight mg, Coulomb repulsion F_e

Equilibrium:

$$T\cos\alpha = mg \qquad T\sin\alpha = F_e \qquad \Rightarrow \qquad \tan\alpha = \frac{F_e}{mg}$$

(c) Charge on Each Ball

Mass:

$$V = \tfrac{4}{3}\pi(0.005)^3 = 5.236 \times 10^{-7}\ \text{m}^3, \qquad m = 2700 \times 5.236 \times 10^{-7} = 1.414 \times 10^{-3}\ \text{kg}$$

Separation:

$$r = 2L\sin\alpha = 2 \times 0.10 \times \sin 10^\circ = 3.472 \times 10^{-2}\ \text{m}$$

Coulomb force from equilibrium:

$$F_e = mg\tan\alpha = 1.414 \times 10^{-3} \times 9.8 \times \tan 10^\circ = 2.444 \times 10^{-3}\ \text{N}$$

Charge:

$$q^2 = \frac{F_e r^2}{k} = \frac{2.444 \times 10^{-3} \times (3.472 \times 10^{-2})^2}{9 \times 10^9} = 3.27 \times 10^{-16}\ \text{C}^2$$

$$\boxed{q = 1.81 \times 10^{-8}\ \text{C} = 18.1\ \text{nC}}$$

(d) New Charge on the Metal Disk

When the disk contacts both balls, charge is shared equally among 3 identical conductors:

$$Q_{initial} = 3q \implies Q_{disk,\ remaining} = q$$

$$\boxed{Q_{disk} = 1.81 \times 10^{-8}\ \text{C} = 18.1\ \text{nC}}$$

(e) Gravitational Force Between the Balls

$$F_g = G\frac{m^2}{r^2} = 6.674 \times 10^{-11} \times \frac{(1.414 \times 10^{-3})^2}{(3.472 \times 10^{-2})^2} = 6.674 \times 10^{-11} \times 1.659 \times 10^{-3}$$

$$\boxed{F_g = 1.107 \times 10^{-13}\ \text{N}}$$

(f) Electrostatic Force Between the Balls

$$F_e = k\frac{q^2}{r^2} = 9 \times 10^9 \times \frac{(1.81 \times 10^{-8})^2}{(3.472 \times 10^{-2})^2} = 9 \times 10^9 \times 2.719 \times 10^{-13}$$

$$\boxed{F_e = 2.447 \times 10^{-3}\ \text{N} \approx 2.45\ \text{mN}}$$

Consistent with equilibrium result $F_e = mg\tan\alpha$ ✓

(g) Ratio of Forces

$$\frac{F_e}{F_g} = \frac{2.447 \times 10^{-3}}{1.107 \times 10^{-13}}$$

$$\boxed{\frac{F_e}{F_g} \approx 2.21 \times 10^{10}}$$

The electrostatic force is **22 billion times** greater than gravity between the balls.

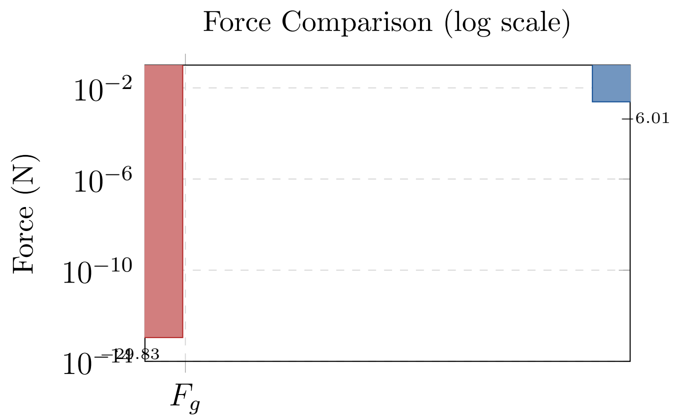

Figure 3: Gravitational vs. electrostatic force (logarithmic scale)

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Given: $q_A = +6\ \mu\text{C}$, $q_B = -2\ \mu\text{C}$, $d = 0.250$ m apart, $q_C = -4\ \mu\text{C}$.

Region Analysis

Since q_C is negative: q_A attracts it, q_B repels it.

Region	**Force from q_A**	**Force from q_B**	$\vec{E} = 0$**?**
Left of q_A	rightward	leftward	No ($\vec{E} \neq 0$)
Between q_A and q_B	rightward	rightward	No (same direction)
Right of q_B	**leftward**	**rightward**	**Yes ✓**

For opposite-sign charges, $\vec{E} = 0$ only exists **outside**, on the side of the weaker charge. Since $\vec{F}_{net} = q_C\vec{E}_{net} = 0$ and $q_C \neq 0$, we need $\vec{E}_{net} = 0$.

Finding the Exact Position

Let x = distance to the right of q_B:

$$\frac{k|q_A|}{(d+x)^2} = \frac{k|q_B|}{x^2} \implies \frac{6}{(0.250+x)^2} = \frac{2}{x^2} \implies 3x^2 = (0.250+x)^2$$

$$\sqrt{3}\,x = 0.250 + x \implies x(\sqrt{3}-1) = 0.250$$

$$x = \frac{0.250}{\sqrt{3}-1} = \frac{0.250}{0.7321} \qquad \boxed{x = 0.3415 \text{ m to the right of } q_B}$$

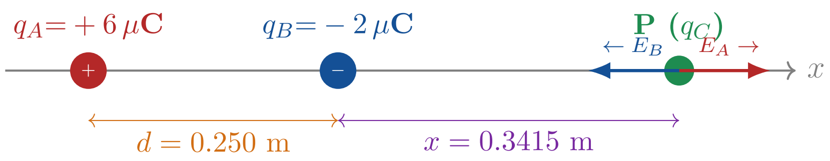
Electric Field Verification

$$E_A = \frac{9\times10^9 \times 6\times10^{-6}}{(0.5915)^2} = 1.543\times10^5 \text{ N/C} \quad \rightarrow$$

$$E_B = \frac{9\times10^9 \times 2\times10^{-6}}{(0.3415)^2} = 1.544\times10^5 \text{ N/C} \quad \leftarrow$$

Equal magnitudes, opposite directions ✓

$$\boxed{\vec{E}_{net} = 0 \text{ N/C at point P}}$$



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Given: $q_1 = q_2 = +6$ nC at $x_A = -10$ cm, $x_B = +10$ cm; $q_3 = -6$ nC at origin; $q_4 = +10$ nC free on y-axis. Gravity neglected.

1. Free Body Diagram at P$(0, y)$

By symmetry, q_1 and q_2 are equidistant from P — horizontal components cancel, vertical components add.

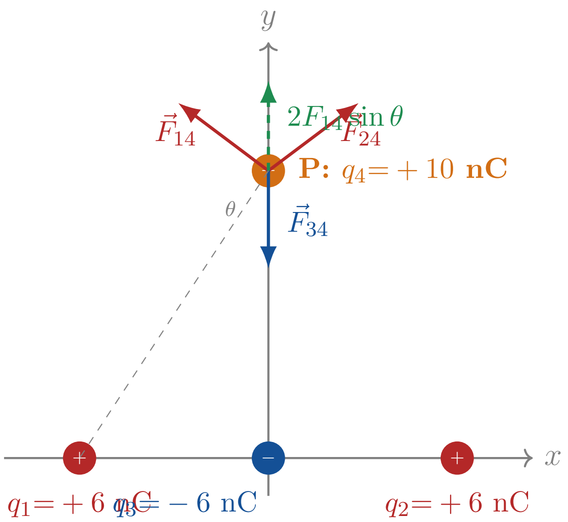

Figure 4: Forces on q_4 at P$(0, y)$. Horizontal components of $\vec{F}_{14}$ and $\vec{F}_{24}$ cancel by symmetry.

Net force condition:

$$\vec{F}_{net} = (2F_{14}\sin\theta - F_{34})\,\hat{j}$$

2. Equilibrium Position y_0

With $r_1 = \sqrt{y^2 + d^2}$, $d = 0.10$ m, $\sin\theta = y/r_1$:

$$2 \cdot k\frac{q_1 q_4}{r_1^2} \cdot \frac{y}{r_1} = k\frac{|q_3| q_4}{y^2} \implies \frac{2y}{(y^2 + d^2)^{3/2}} = \frac{1}{y^2}$$

$$2y^3 = (y^2 + d^2)^{3/2} \implies (2y^3)^{2/3} = y^2 + d^2 \implies 2^{2/3} y^2 = y^2 + d^2$$

$$y^2(2^{2/3} - 1) = d^2 \implies y^2 = \frac{(0.10)^2}{0.5874} = 0.01703 \text{ m}^2$$

$$\boxed{y_0 = \pm 0.1305 \text{ m} = \pm 13.05 \text{ cm}}$$

3. Net Electric Field at P

Since $\vec{F}_{net} = q_4\vec{E}_{net} = 0$ and $q_4 \neq 0$:

$$\boxed{\vec{E}_{net} = 0 \text{ N/C}}$$

Verification: $r_1 = \sqrt{(0.1305)^2 + (0.10)^2} = 0.1644$ m

$$E_1 = E_2 = \frac{9 \times 10^9 \times 6 \times 10^{-9}}{(0.1644)^2} = 1997 \text{ N/C}$$

$$2E_1 \sin\theta = 2 \times 1997 \times \frac{0.1305}{0.1644} = 3171 \text{ N/C} \uparrow$$

$$E_3 = \frac{9 \times 10^9 \times 6 \times 10^{-9}}{(0.1305)^2} = 3171 \text{ N/C} \downarrow$$

Equal and opposite ✓

4. Electric Potential at P

$$V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right) = 9 \times 10^9 \left(\frac{6 \times 10^{-9}}{0.1644} + \frac{6 \times 10^{-9}}{0.1644} + \frac{-6 \times 10^{-9}}{0.1305}\right)$$
$$= 9 \times 10^9 \left(72.99 \times 10^{-9} - 45.98 \times 10^{-9}\right) = 9 \times 10^9 \times 27.01 \times 10^{-9}$$

$$\boxed{V = 243.1 \text{ V}}$$

5. Work Done by External Agent

$$W_{ext} = q_4(V_P - V_\infty) = q_4V_P = 10 \times 10^{-9} \times 243.1$$

$$\boxed{W_{ext} = 2.431\ \mu\text{J}}$$

6. Potential Energy of q_4

$$\boxed{U_{q_4} = q_4V_P = 2.431\ \mu\text{J}}$$

7. Total Energy of the System

All 6 unique pairs:

Pair	**Charges (nC)**	**Distance**	**Energy (μJ)**
q_1–q_2	$(+6)(+6)$	0.20 m	$+1.620$
q_1–q_3	$(+6)(-6)$	0.10 m	-3.240
q_2–q_3	$(+6)(-6)$	0.10 m	-3.240
q_1–q_4	$(+6)(+10)$	0.1644 m	$+3.285$
q_2–q_4	$(+6)(+10)$	0.1644 m	$+3.285$
q_3–q_4	$(-6)(+10)$	0.1305 m	-4.138
Total			-2.428

$$\boxed{U_{total} = -2.428\ \mu\text{J}}$$

Negative total energy: the system is in a **bound state** — energy was released during assembly.

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Given: $Q_A = Q_B = -q$, $Q_C = +q$, equilateral triangle side d.
Coordinates:

$$A = \left(-\tfrac{d}{2}, 0\right), \quad B = \left(+\tfrac{d}{2}, 0\right), \quad C = \left(0, \tfrac{d\sqrt{3}}{2}\right)$$

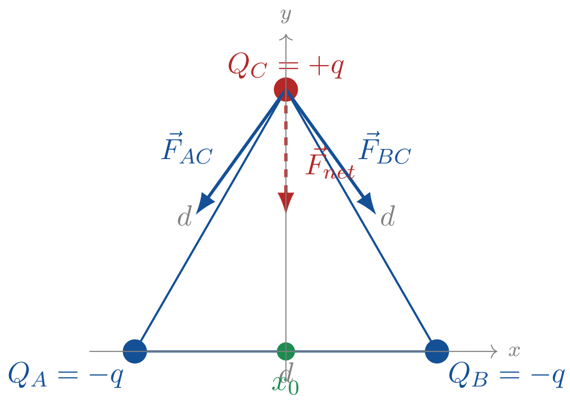

Figure 5: Equilateral triangle: forces on Q_C and midpoint x_0

(1) Force on Q_C

By symmetry, horizontal components cancel:

$$|\vec{F}_{AC}| = |\vec{F}_{BC}| = k\frac{q^2}{d^2}$$

Vertical components:

$$F_{net,y} = -2 \cdot k\frac{q^2}{d^2} \cdot \sin 60 = -2 \cdot k\frac{q^2}{d^2} \cdot \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}\,kq^2}{d^2}$$

$$\boxed{|\vec{F}_{net}| = \frac{\sqrt{3}\,kq^2}{d^2}, \quad \text{directed straight downward } (-\hat{y})}$$

Toward the midpoint of Q_AQ_B (along symmetry axis).

(2) Electric Field at Midpoint $x_0 = (0, 0)$

Distances: $r_A = r_B = d/2$, $r_C = d\sqrt{3}/2$.

$$|\vec{E}_A| = |\vec{E}_B| = k\frac{q}{(d/2)^2} = \frac{4kq}{d^2}$$

$\vec{E}_A$ points toward Q_A (leftward), $\vec{E}_B$ points toward Q_B (rightward) $\Rightarrow$ **cancel**.

$$|\vec{E}_C| = k\frac{q}{(d\sqrt{3}/2)^2} = k\frac{q}{3d^2/4} = \frac{4kq}{3d^2}$$

Q_C is positive $\Rightarrow$ field at origin points **away** from Q_C = downward $(-\hat{y})$.

$$\boxed{\vec{E}_{net} = \frac{4kq}{3d^2}\,(-\hat{y}), \quad \text{directed straight downward}}$$

(3) Potential Energy $U(y)$ and Motion of Q_C

General potential energy for Q_C at position $(0, y)$:

$$U(y) = k\frac{Q_A Q_C}{r_{AC}} + k\frac{Q_B Q_C}{r_{BC}} = -\frac{2kq^2}{\sqrt{y^2 + d^2/4}}$$

$$\boxed{U(y) = -\frac{2kq^2}{\sqrt{y^2 + d^2/4}}}$$

Energy at key positions:

Position	$U(y)$	KE
$y_0 = +\frac{d\sqrt{3}}{2}$ (start, rest)	$-\frac{2kq^2}{d}$	0
$y = 0$ (midpoint, max speed)	$-\frac{4kq^2}{d}$	$+\frac{2kq^2}{d}$
$y = -\frac{d\sqrt{3}}{2}$ (turning point)	$-\frac{2kq^2}{d}$	0

Description of motion:

1. Released from rest at apex $C = (0,\ d\sqrt{3}/2)$
2. Net force is **downward** $\Rightarrow$ Q_C accelerates toward midpoint
3. Reaches **maximum kinetic energy** $KE_{max} = \frac{2kq^2}{d}$ at $y = 0$
4. Overshoots — now force reverses (upward) $\Rightarrow$ decelerates
5. Comes to rest at symmetric turning point $y = -d\sqrt{3}/2$
6. Returns upward $\Rightarrow$ **oscillates indefinitely** between $\pm\, d\sqrt{3}/2$

The motion is **symmetric oscillation along the y-axis**, driven by the attractive Coulomb forces of Q_A and Q_B.

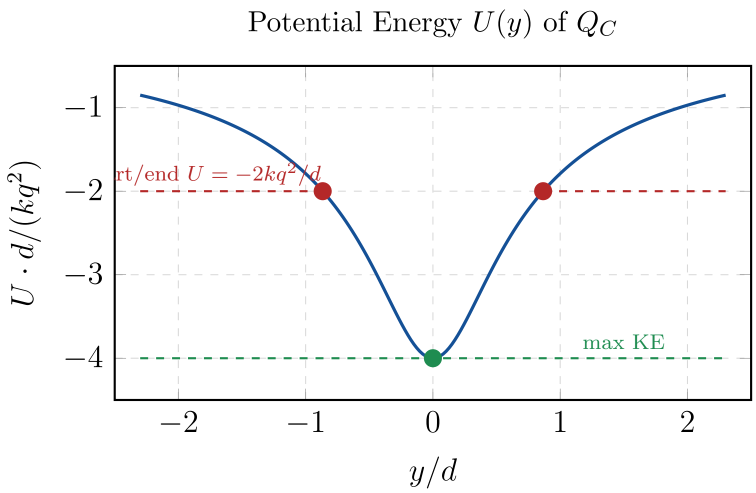

Figure 6: Potential energy profile: Q_C oscillates between the two red points, with maximum speed at the green point ($y = 0$).

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Exercise	**Key Result**	**Value**
Ex.1 Glass & Silk	Charge transferred	$Q = \pm 4.8 \times 10^{-8}$ C
Ex.2 Electrophorus	Charge per ball Disk charge remaining Gravitational force Electrostatic force Force ratio	18.1 nC 18.1 nC 1.107×10^{-13} N 2.447×10^{-3} N 2.21×10^{10}
Ex.3 Three Charges	Position of q_C Electric field at P	0.3415 m right of q_B 0 N/C
Ex.4 Four Charges	Equilibrium y_0 $\vec{E}$ at P Potential V Work W_{ext} U_{q_4} U_{total}	± 13.05 cm 0 N/C 243.1 V 2.431 μJ 2.431 μJ -2.428 μJ
Ex.5 Triangle	Force on Q_C Field at x_0 $U(y)$ Motion	$\sqrt{3}kq^2/d^2 \ \downarrow$ $4kq/(3d^2) \ \downarrow$ $-2kq^2/\sqrt{y^2 + d^2/4}$ Oscillation $\pm d\sqrt{3}/2$