Fluid Mechanics
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Exercise Set 2: Statics of Fluids (Hydrostatics)

Fluid Mechanics — Chapter 2

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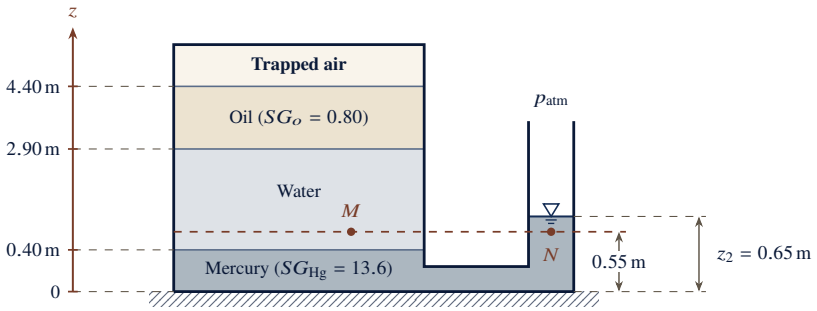
Useful Data

Water density: $\rho_w = 1000\,\text{kg m}^{-3}$; gravitational acceleration: $g = 9.81\,\text{m s}^{-2}$; standard atmospheric pressure: $p_{\text{atm}} = 101.325\,\text{kPa}$; specific gravity of mercury: $SG_{\text{Hg}} = 13.6$; universal gas constant: $R = 8.314\,\text{J mol}^{-1}\,\text{K}^{-1}$. Unless stated otherwise, all liquids are incompressible and homogeneous, and $SG = \rho/\rho_w = \gamma/\gamma_w$ with $\gamma = \rho g$.

Exercise 1

A closed cylindrical tank is connected at its flat horizontal floor ($z = 0$) to an open vertical standpipe, as shown below. Mercury ($SG_{\text{Hg}} = 13.6$) occupies the bottom of the tank up to $z_1 = 0.40\,\text{m}$ and rises in the open standpipe to $z_2 = 0.65\,\text{m}$. Inside the closed tank above the mercury sit a water layer of thickness $h_w = 2.50\,\text{m}$ (from $z = 0.40\,\text{m}$ to $2.90\,\text{m}$), an oil layer ($SG_o = 0.80$) of thickness $h_o = 1.50\,\text{m}$ (from $z = 2.90\,\text{m}$ to $4.40\,\text{m}$), and a pocket of trapped air at the top.



1. Determine the gauge pressure and the absolute pressure of the trapped air, and express the absolute air pressure as an equivalent mercury head (in mmHg).
2. Compute the absolute pressure acting on the tank floor ($z = 0$).
3. Points M (inside the water layer) and N (inside the standpipe mercury column) lie at the same elevation $z_M = z_N = 0.55\,\text{m}$. State whether $p_M = p_N$, justify your answer, and compute $p_M - p_N$.

Exercise 2

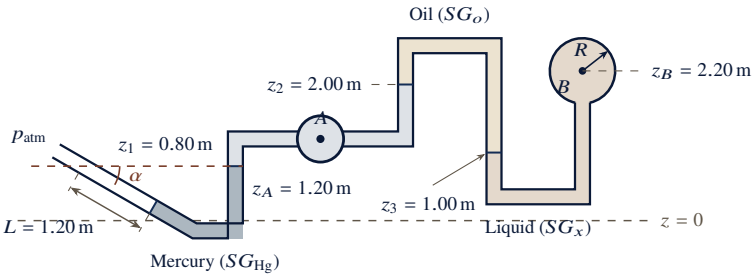
Atmospheric air is modeled as an ideal gas of molar mass $M = 29.0\,\text{g mol}^{-1}$. At sea level ($z_0 = 0$), the pressure and temperature are $p_0 = 101.325\,\text{kPa}$ and $T_0 = 288\,\text{K}$. Assume gravitational acceleration g remains constant over the altitude range considered.

1. Starting from the fundamental equation of fluid statics and the ideal-gas equation of state, derive the pressure distribution $p(z)$ as a function of altitude z for an isothermal atmosphere at temperature T_0.
2. Compute the atmospheric pressure and air density at altitude $z_1 = 4000\,\text{m}$.
3. Recompute the pressure at $z_1 = 4000\,\text{m}$ under the assumption of an incompressible atmosphere having uniform sea-level density ρ_0, and determine the resulting relative error (in %).
4. Determine the mercury column height h (in mm) indicated by a mercury barometer of negligible mercury vapor pressure at altitude $z_1 = 4000\,\text{m}$.

Exercise 3

A water suction pipe centered at A ($z_A = 1.20\,\text{m}$) is connected simultaneously to two manometric branches, as illustrated below:

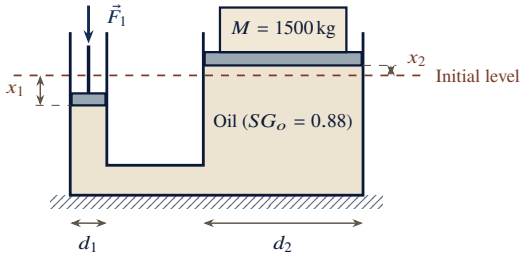
- **Left branch:** a mercury U-tube ($SG_{\text{Hg}} = 13.6$) whose outer leg is inclined at $\alpha = 30°$ to the horizontal and open to the atmosphere. The water–mercury interface in the vertical leg sits at $z_1 = 0.80\,\text{m}$, while the open mercury meniscus lies at a slant distance $L = 1.20\,\text{m}$ below the horizontal plane $z = z_1$ measured along the inclined tube.
- **Right branch:** an inverted U-tube containing oil ($SG_o = 0.80$) in its upper bend, with the water–oil interface at $z_2 = 2.00\,\text{m}$ and the right oil interface at $z_3 = 1.00\,\text{m}$. Beyond z_3, the tube leads into a closed spherical bulb of radius $R = 0.30\,\text{m}$ centered at B ($z_B = 2.20\,\text{m}$), completely filled with an unknown liquid of specific gravity SG_x and negligible vapor pressure.



1. Determine the gauge pressure and the absolute pressure at the pipe centerline A.
2. Determine the specific gravity SG_x of the liquid filling the spherical bulb.
3. Compute the error committed on SG_x if the zero-pressure condition inside the bulb were imposed at the sphere center z_B rather than at the highest point of the liquid.

Exercise 4

A hydraulic lift filled with oil ($SG_o = 0.88$) connects two vertical cylinders of diameters $d_1 = 5.0\,\text{cm}$ and $d_2 = 25.0\,\text{cm}$, fitted with tight frictionless pistons of negligible mass. Piston 2 supports a vehicle of total mass $M = 1500\,\text{kg}$, and atmospheric pressure acts on the outer faces of both pistons.

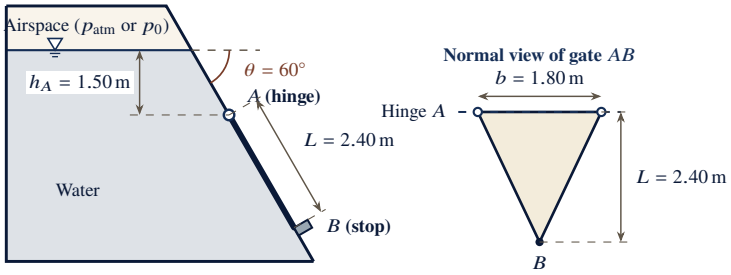


1. When the lower faces of both pistons lie at the same horizontal elevation, determine the vertical force F_0 that must be applied to piston 1 to hold the vehicle in equilibrium.
2. Piston 1 is pushed downward by $x_1 = 25.0\,\text{cm}$ from the initial level. Express the upward displacement x_2 of piston 2 in terms of x_1, d_1, and d_2, and compute its numerical value.
3. Determine the vertical force F_1 required on piston 1 to maintain equilibrium in this displaced configuration.

Exercise 5

A reservoir wall inclined at $\theta = 60°$ to the horizontal contains a rigid planar isosceles triangular gate AB of horizontal top base $b = 1.80\,\text{m}$ and slant altitude $L = 2.40\,\text{m}$. The gate is hinged without friction along its horizontal top base A at vertical depth $h_A = 1.50\,\text{m}$ below the water surface, and its bottom vertex B rests against

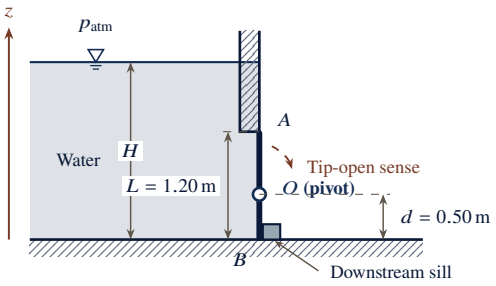
a smooth stop exerting a normal reaction R_B perpendicular to the gate. The outer face of the gate is exposed to atmospheric air, and the gate weight is negligible.



1. Assuming the airspace above the water is open to the atmosphere, determine the magnitude F_R of the resultant hydrostatic force exerted by the water on the gate, the slant distance from hinge A to the center of pressure, and the stop reaction R_B.
2. The reservoir roof is now sealed and the trapped airspace is pressurized to a gauge reading $p_0 = 19.62\,\text{kPa}$ while the outer face of the gate remains at atmospheric pressure. Determine the new resultant net force on the gate, the slant distance from hinge A to its point of application, and the new stop reaction R'_B.

Exercise 6

A vertical rectangular floodgate AB of height $L = 1.20\,\text{m}$ (from the horizontal floor $z = 0$ at B to $z = L$ at A) and width $b = 2.00\,\text{m}$ (perpendicular to the figure) closes an opening at the base of a vertical dam wall. The gate pivots without friction about a horizontal shaft O located at height $d = 0.50\,\text{m}$ above the floor. A fixed sill on the downstream side of the bottom edge B prevents the lower portion OB from rotating outward, while allowing the gate to tip open when the upper portion OA swings outward (away from the reservoir) and OB swings inward. The reservoir is open to the atmosphere and holds water to variable depth $H \geq L$; the downstream side of the gate is exposed to atmospheric air, and the gate mass is negligible.

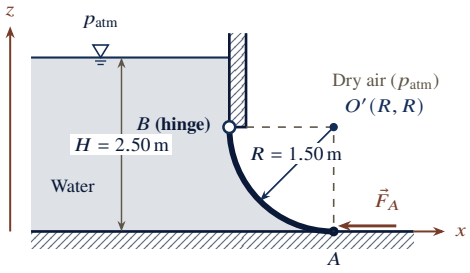


1. For $H = L = 1.20\,\text{m}$, determine the resultant hydrostatic force on the gate, the height z_p of the center of pressure above the floor, and the horizontal reaction R_B exerted by the sill at B. Verify that the gate remains closed.
2. For $H > L$, express the height $z_p(H)$ of the center of pressure above the floor in terms of H and L, and determine the critical water depth H_c above which the gate automatically tips open.
3. If the pivot shaft O were placed at $d' = 0.65\,\text{m}$ above the floor instead of 0.50 m, determine whether the gate could ever tip open for any finite water depth H. Justify your answer.

Exercise 7

An open reservoir holds water to a total depth $H = 2.50\,\text{m}$ above a horizontal floor ($z = 0$). From the free surface down to elevation $z = R = 1.50\,\text{m}$, the right retaining wall is vertical ($x = 0$). Between $B(0, R)$ and $A(R, 0)$ on the floor, the reservoir is closed by a rigid quarter-circular cylindrical gate BA of radius $R = 1.50\,\text{m}$ and width $b = 2.00\,\text{m}$ (perpendicular to the figure) whose center of curvature $O'(R, R)$ lies on the dry external side. The gate

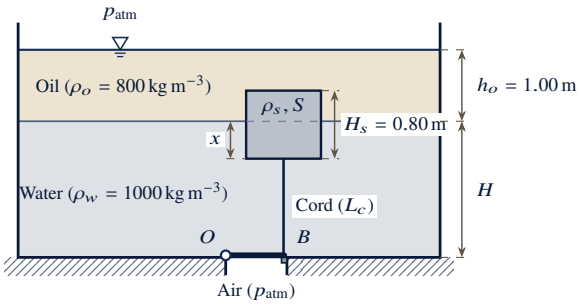
has negligible weight, is hinged without friction along B, and is held closed at its bottom edge A by a horizontal stop exerting a leftward force F_A.



1. Determine the magnitude F_H of the horizontal component of the hydrostatic force on gate BA and the depth h_p of its line of action below the free surface.
2. Determine the magnitude and vertical direction (upward or downward) of the vertical component F_V of the hydrostatic force on gate BA, then deduce the magnitude of the resultant force F_R and its angle with the horizontal.
3. Determine the horizontal force F_A required at A to hold the gate closed.

Exercise 8

An open tank contains a lower water layer ($\rho_w = 1000\,kg\,m^{-3}$) of variable depth H topped by an immiscible upper oil layer ($\rho_o = 800\,kg\,m^{-3}$) of constant thickness $h_o = 1.00\,m$. A homogeneous solid vertical cylinder of cross-sectional area $S = 0.55\,m^2$, height $H_s = 0.80\,m$, and density $\rho_s = 900\,kg\,m^{-3}$ is connected from the center of its lower face by a vertical inextensible cord of length $L_c = 1.00\,m$ to the midpoint of the free edge B of a horizontal rectangular drain gate OB of area $A_v = 0.020\,m^2$. Gate OB is hinged without friction along its opposite edge O on the tank floor ($z = 0$), rests on a narrow recessed floor lip at B (so it can only open upward into the tank), has negligible mass, and is exposed to atmospheric air on its underside.



1. While the cord is slack ($T = 0$) and the cylinder floats completely submerged across the oil–water interface, determine the equilibrium height x_0 of the cylinder immersed in the water layer, and deduce the water depth H_0 at which the cord first becomes taut.
2. For $H \in [H_0,\ L_c + H_s]$ with the drain gate still closed, express the cord tension $T(H)$ in terms of ρ_w, ρ_o, S, g, H, and H_0.
3. Determine the critical water depth H_c at which the drain gate OB automatically opens, and compute the cord tension at that instant.
4. If the cylinder cross-sectional area is reduced to $S' = 0.25\,m^2$ (all other parameters remaining unchanged), determine whether the drain gate ever opens for any water depth $H > 0$. Justify quantitatively.