



## discrete random variable set

### Exercise 1:

We consider the 140 students present in the lecture theatre:

- 1) A student is chosen at random from the lecture theatre. What is the probability that his birthday will be on a Monday?
- 2) Students are asked one by one if their birthday falls on Monday. What is the probability that the seventh student is the first to answer "YES"?
- 3) Let  $X$  be the r.v. giving the number of students whose next birthday is on Monday.
  - a) Give the distribution function of  $X$ .
- 4) Compute  $P(X = 20)$ .

### Exercise 2:

In a certain population, 2% of people suffer from respiratory disorders. A sample of  $n$  individuals is chosen at random with replacement. What should the size of this sample be, if we want to be more than 95% certain that it will contain at least one individual suffering from respiratory disorders?

### Exercise 3:

The after-sales service of a hypermarket specializing in computer equipment has teams that intervene on customer calls. Interventions sometimes occur with delay. It is assumed that calls occur independently of each other and that for each call, the probability of a delay is 0.25.

- 1) A customer calls the service eight times. Let  $X$  denote the number of times this customer experienced a delay.
  - a) Determine the distribution, expectation, and variance of  $X$ .
  - b) Calculate the probability of the event "the customer experienced at least one delay."
  - c) Calculate the probability of the event "the customer experienced fewer than four delays."
  - d) Calculate the probability of the event "the customer experienced at least one."

**Exercise 4:**

There is a 0.9986 probability that a randomly selected 30-year-old male lives through the year. A life insurance company charges \$161 for insuring that the male will live through the year. If the male does not survive the year, the policy pays out \$100,000 as a death benefit. Assume that the company sells 1300 such policies so it collects \$209,300 in policy payments. The company will make a profit if the number of deaths in this group is 2 or fewer.

- 1) What is the mean number of deaths in such groups of 1300 males?
- 2) Use the Poisson distribution to find the probability that the company makes a profit from the 1300 policies.
- 3) Use the binomial distribution to find the probability that the company makes a profit from the 1300 policies, then compare the result to part (2).

**Exercise 5:**

A hospital receives an average of 12 emergency calls per day. The probability that an emergency call is for a heart attack is 0.2. Assume that the number of emergency calls and the type of emergency are independent.

- 1) What is the probability that the hospital receives exactly 3 emergency calls in one day?
- 2) What is the average number of emergency calls per week, and what is the probability that the hospital receives at least 5 emergency calls in one week?
- 3) What is the probability that the hospital receives at least 5 emergency calls for heart attacks among the average of all emergency calls in one week?
- 4) What is the probability that the hospital receives no emergency calls for heart attacks in two days?

**Exercise 6: Mushroom Picking:**

- 1) Mohamed goes mushroom picking. He does not know how to distinguish between edible and poisonous mushrooms. It is estimated that the proportion of poisonous mushrooms in the forest is 0.7.
  - a) Mohamed picks 6 mushrooms at random. Calculate the probability that he picks exactly 4 poisonous mushrooms.
  - b) Mohamed invites Ali to go picking. Ali knows mushrooms well; out of 10 mushrooms he picks, 9 are edible. That day, he picks 4 mushrooms and Mohamed picks 3 mushrooms. Calculate the probability that all the mushrooms picked are edible.
- 2) Ali picks on average 12 mushrooms per hour.
  - a) Calculate the probability that he picks exactly 8 mushrooms in one hour.
  - b) Calculate the probability that he picks at least 1 mushroom in 20 minutes.

**Exercise 7:**

At a post office, counter number 1 handles two types of operations: cash withdrawals and deposits of registered items. We make the following assumptions: during any time interval of length  $T$  minutes within opening hours, the number of people arriving at counter 1 for cash withdrawals is represented by a real random variable  $X_a$ , following a Poisson distribution with parameter  $aT$  (with  $a > 0$ ), and the number of people arriving for deposits is represented by a real random variable  $X_b$ , following a Poisson distribution with parameter  $bT$ . The random variables  $X_a$  and  $X_b$  are independent.

- 1) Show that the probability distribution of the random variable representing the total number of people arriving at counter 1 follows a Poisson distribution with parameter  $(a+b)T$ . If  $a = 0.4$  and  $b = 0.2$ , what is the probability that no customer arrives between 10:00 and 10:05?
- 2) Since employees must finish work at 19:00, the post office closes at 18:45. At 18:45, there is a queue of  $n$  people at counter 1, who all arrived between 18:35 and 18:45. Express, in terms of  $a$ ,  $b$ , and  $n$ , the probability distribution of the number of people in this queue who came for cash withdrawals.  
For  $n = 5$ ,  $a = 0.4$ , and  $b = 0.2$ , compute the probability that in this queue, the number of people who came for cash withdrawals is less than the number of people who came for deposits.

**Exercise 8:**

(Banach Matchbox Problem) A person always carries two boxes of matches, one in the left pocket and one in the right pocket. Each time they need a match, they choose one of the two boxes at random. They suddenly discover that the chosen box is empty. Both boxes initially contained  $n$  matches. What is the probability that there are  $k$  matches left in the other box? For  $n = 6$ , what is the expected number of matches remaining in the non-empty box?

**Exercise 9:**

A clinic receives 6 road accident victims every night, chosen at random from all victims in the region.

- 1) Knowing that out of a large number of road accident victims the probability of having a serious case is 0.2, calculate the probability that the clinic receives more than one serious case each night.
- 2) If more than one serious case occurs during the night, Dr. Ahmed is called to handle all the cases.
  1. What is the probability that Dr. Ahmed is called?
  2. What is the probability that, during one week, Dr. Ahmed is disturbed at least one night?
  3. What is the probability that he is disturbed more than a third of his nights knowing that he works only 12 days per month?