Exercise Series

Joint Random Variables

Course: Probability & Statistics
Level: Undergraduate
Additional Exercises

Exercise 1 — Discrete Random Variables

Let (X, Y) be a pair of random variables taking values in $\mathbb{N} \times \mathbb{N}$, with probability law

$$P(X = i, Y = j) = \begin{cases} \dfrac{e^{-1}}{2^{i+1} j!}, & (i, j) \in \mathbb{N} \times \mathbb{N}, \\ 0, & \text{otherwise.} \end{cases}$$

1. Verify that the relation above defines a probability law on $\mathbb{N} \times \mathbb{N}$.
2. Determine the marginal probability law of X, then that of Y.
3. Are X and Y independent? Justify your answer.
4. Determine the conditional probability law of Y given $X = i$, where i is a nonnegative integer.

Exercise 2 — Discrete Random Variables

Let X and Y be two independent random variables such that

$$P(X = -1) = \frac{1}{8}, \qquad P(X = 0) = \frac{1}{2}, \qquad P(X = 1) = \frac{3}{8},$$

and

$$P(Y = 0) = \frac{3}{8}, \qquad P(Y = 1) = \frac{5}{8}.$$

1. Determine the cumulative distribution function F of X.
2. Calculate $P(-X^2 + 1 > 0)$ and $P(|X| \geq 1)$.
3. Calculate $E(X)$ and $\mathrm{Var}(X)$.
4. Determine the probability law of $Z = X + Y$.

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