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Exercise Set 1: DC Networks, Sinusoidal Steady State, Phasors & AC Power

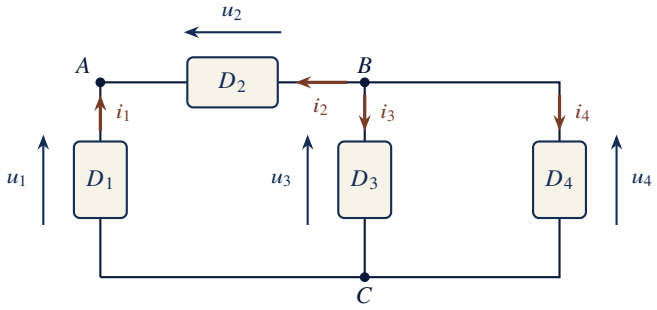
General Electricity — Chapter 1

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Conventions & Complex Phasor Notation For a potential difference $u_{AB} = V_A - V_B$, the voltage arrow points toward terminal A. In the sinusoidal steady state ($j^2 = -1$), $\bar{U} = Ue^{j\varphi_u} = U\angle\varphi_u$ and $\bar{I} = Ie^{j\varphi_i} = I\angle\varphi_i$ denote RMS complex phasors ($U = U_m/\sqrt{2}$, $I = I_m/\sqrt{2}$), and $\vec{U}(U, \varphi_u)$, $\vec{I}(I, \varphi_i)$ denote the associated RMS Fresnel vectors.

Exercise 1

Four two-terminal dipoles D_1, D_2, D_3, and D_4 are interconnected between nodes A, B, and C in the DC network shown below. With the reference voltage and current arrows indicated on the diagram, measurements give $u_1 = 28\,\text{V}$, $u_2 = 12\,\text{V}$, $i_1 = 5\,\text{A}$, and $i_3 = 8\,\text{A}$.



1) Determine the voltages u_3 and u_4 and the algebraic currents i_2 and i_4.

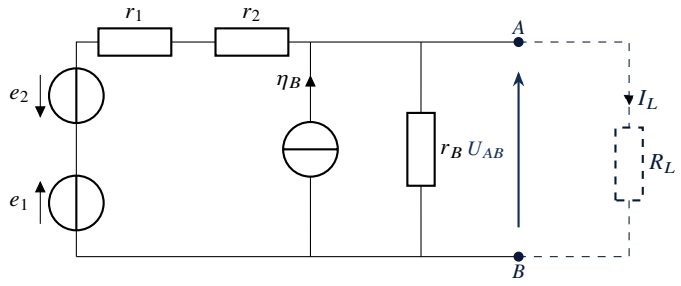
2) For each dipole D_k ($k = 1, 2, 3, 4$), identify whether its labeled arrows follow the receiver convention or the generator convention, compute $p_k = u_k\, i_k$, and state whether D_k absorbs or supplies electrical power.

3) Dipole D_4 is re-labeled in the generator convention while keeping $u_4 = V_B - V_C$ unchanged. Give the new reference current i_4', compute $p_{4,\text{gen}} = u_4\, i_4'$, and compare the physical conclusion with question 2.

4) Verify power conservation across the complete network.

Exercise 2

In the DC generator network shown below, Branch I contains two real voltage generators ($e_1 = 30\,\text{V}$, $r_1 = 2.5\,\Omega$) and ($e_2 = 6\,\text{V}$, $r_2 = 1.5\,\Omega$) connected in series opposition, while Branch II is a real current generator ($\eta_B = 2\,\text{A}$, $r_B = 12\,\Omega$) connected in parallel across terminals (A, B).



1) Reduce Branch I to a single real voltage generator (e_A, r_A), then convert it into its equivalent current generator (η_A, r_A).

2) Deduce the single equivalent current generator $(\eta_{\text{eq}}, r_{\text{eq}})$ and the single equivalent voltage generator $(e_{\text{eq}}, r_{\text{eq}})$ of the entire active network left of terminals (A, B). Give the open-circuit voltage U_{oc} and the short-circuit

current I_{sc} across (A, B).

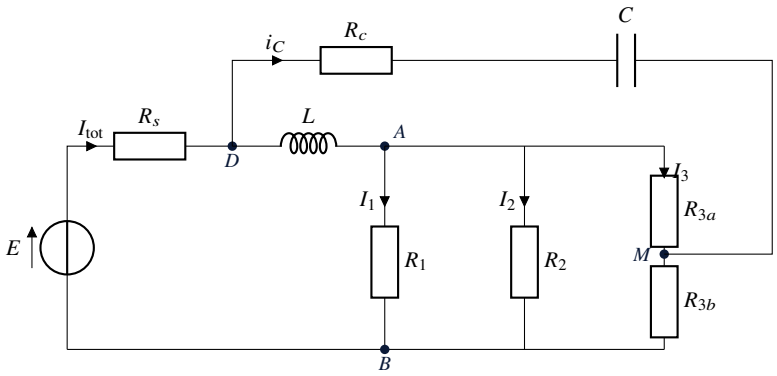
3) Write the external characteristic $U_{AB}(I_L)$ and determine the operating point (I_L, U_L) and load power P_L when $R_L = 5\,\Omega$ is connected across (A, B), checking that both equivalent models yield identical results.

4) With (A, B) left in open circuit ($I_L = 0$), compute the internal Joule power dissipated inside the single equivalent voltage model (e_{eq}, r_{eq}) and inside the single equivalent current model (η_{eq}, r_{eq}). Comment.

5) Explain why an ideal voltage source ($r = 0\,\Omega$) cannot be transformed into an equivalent current source.

Exercise 3

Consider the circuit below operating in the **DC steady state**, with $R_s = 4\,\Omega$, $R_1 = 6\,\Omega$, $R_2 = 12\,\Omega$, $R_{3a} = 1\,\Omega$, $R_{3b} = 3\,\Omega$, $R_c = 7\,\Omega$, an ideal coil $L = 50\,\text{mH}$, and an ideal capacitor $C = 20\,\mu\text{F}$. An ideal ammeter inserted in the middle parallel branch indicates $I_2 = 2.0\,\text{A}$.



1) State the behavior of the ideal coil L and ideal capacitor C in the DC steady state, and give the values of u_L and i_C.

2) Calculate the equivalent conductance G_p and equivalent resistance R_p of the three-branch parallel block between nodes A and B.

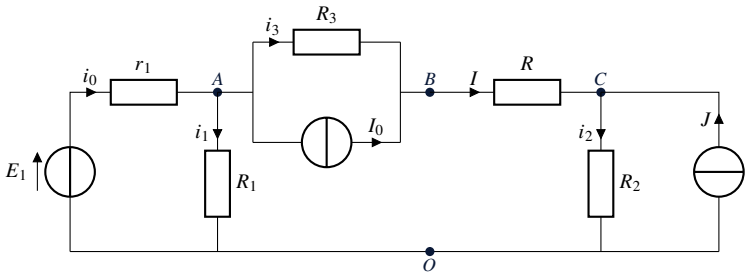
3) From $I_2 = 2.0\,\text{A}$, determine the voltage V_{AB}, the total current I_{tot}, the branch currents I_1 and I_3, and the source electromotive force E. Verify I_1, I_2, and I_3 using the current divider rule.

4) Determine the potential $V_M - V_B$, the steady-state capacitor voltage $u_C = V_D - V_M$ (across the plates of C), and the electric charge q_C stored on C.

5) Which electrical state variables of L and C remain continuous immediately after an abrupt switching operation in the network?

Exercise 4

Part A. Consider Circuit (a) below, where $E_1 = 20\,\text{V}$, $r_1 = 4\,\Omega$, $R_1 = 4\,\Omega$, $R_3 = 3\,\Omega$, $I_0 = 2.0\,\text{A}$, $R = 3\,\Omega$, $R_2 = 6\,\Omega$, and $J = 1.5\,\text{A}$.



Circuit (a)

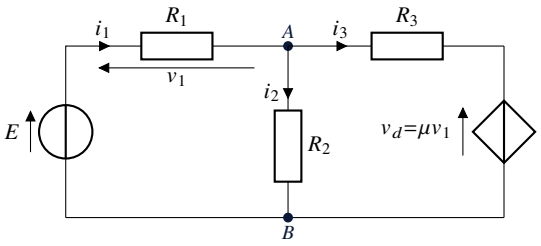
1) Treating (E_1, r_1) as a single branch, count the principal nodes N, the branches B, the number of independent KCL equations, and the number of independent KVL mesh equations required once the ideal current sources fix their branch currents.

2) Find the current I through resistor R using generator equivalences.

3) Write KCL and KVL directly on Circuit (a) to determine all branch currents (i_0, i_1, i_3, I, i_2), verifying agreement with question 2, and deduce the terminal voltage $U_J = V_C - V_O$ across the ideal current source J.

4) Without re-solving the full system, state by how much the KCL current balance at node C shifts if the physical orientation of source J (1.5 A) is reversed while I is held fixed.

Part B. In Circuit (b) below, $E = 18\,\text{V}$, $R_1 = 3\,\Omega$, $R_2 = 4\,\Omega$, $R_3 = 6\,\Omega$, and the diamond element is a controlled source imposing terminal voltage $v_d = \mu\, v_1$ with $\mu = 3$, where $v_1 = R_1\, i_1$ is the ohmic drop across R_1.



Circuit (b)

5) Name the exact category of the controlled source $v_d = \mu\, v_1$ and give the physical dimension of μ.

6) Apply KCL and KVL to determine (i_1, i_2, i_3), the controlled voltage v_d, the power absorbed or supplied by the controlled source, and the equivalent input resistance $R_{\text{in}} = E/i_1$ seen by source E.

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Exercise 5

A passive linear dipole operating in the sinusoidal steady state receives the terminal voltage and current (with t in seconds):

$$u(t) = 120\sqrt{2}\sin\left(100\pi t - \frac{\pi}{4}\right)\ \text{V}, \qquad i(t) = 6\sqrt{2}\cos\left(100\pi t - \frac{5\pi}{12}\right)\ \text{A}.$$

1) Determine the angular frequency ω, frequency f, and period T. Rewrite $i(t)$ in the standard $\sin(\omega t + \varphi_i)$ reference, and express both initial phases φ_u and φ_i in radians and in degrees.

2) By explicit integration over one period T, evaluate the full-period mean voltage $\langle u \rangle$ and establish the RMS value U of $u(t)$.

3) Within the first positive period $t \in [0, T]$, calculate the exact instants of the rising zero-crossing t_{0+}, maximum $t_{\max}$, falling zero-crossing t_{0-}, and minimum $t_{\min}$ of $u(t)$.

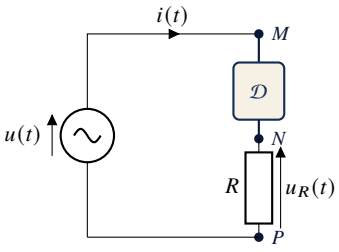
4) Write the RMS complex phasors $\bar{U}$ and $\bar{I}$, and specify the lengths (in cm) and polar angles of the Fresnel vectors $\vec{U}$ and $\vec{I}$ using scales $30\ \text{V cm}^{-1}$ and $2\ \text{A cm}^{-1}$.

5) Calculate the phase shift $\varphi = \varphi_u - \varphi_i$, state which waveform leads the other, and deduce the corresponding time shift $|\Delta t|$ between their rising zero-crossings.

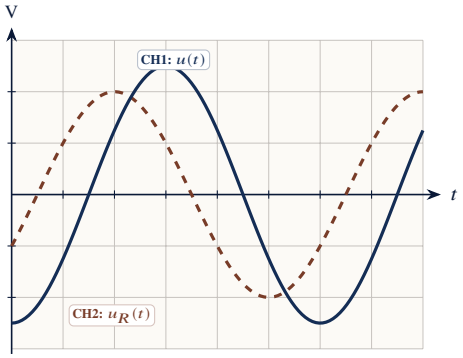
Exercise 6

To characterize an unknown passive linear dipole $\mathcal{D}$ (modeled internally as a resistance r_0 in series with a single reactive element), a series circuit is assembled with a sinusoidal generator $u(t)$ and a precision current-sensing resistor $R = 80\,\Omega$ as shown in Figure (a). A dual-trace oscilloscope records total voltage $u(t)$ on `CH1` and resistor voltage $u_R(t) = R\,i(t)$ on `CH2`, producing the screen display in Figure (b) with settings:

$$S_{V1} = 4\ \text{V/div (CH1)}, \qquad S_{V2} = 2\ \text{V/div (CH2)}, \qquad S_t = 1\ \text{ms/div}.$$



(a) Series test circuit



(b) Dual-trace oscillogram (1 square = 1 div × 1 div)

1) Redraw Figure (a) and indicate at which nodes (M, N, or P) the `CH1` probe, the `CH2` probe, and the common oscilloscope reference ground (`réf.` / ⊥) must be connected.

2) Explain what happens if an operator connects the `CH1` ground clip to node P and the `CH2` ground clip to node N. If $\mathcal{D}$ and R were physically interchanged in Figure (a), what oscilloscope channel function must be activated to display $u_R(t)$ with the proper sign?

3) From Figure (b), determine the period T, frequency f, angular frequency ω, peak voltages U_m and U_{Rm}, RMS voltages U and U_R, and the RMS circuit current I.

4) Determine the horizontal time offset $|\Delta t|$, the signed phase shift $\varphi = \varphi_u - \varphi_i$ (in radians and degrees), and the power factor $\cos\varphi$ of the complete series circuit. Deduce whether $\mathcal{D}$ is inductive or capacitive.

5) Calculate the modulus $|\bar{Z}_{\text{tot}}|$ of the complete circuit impedance, then deduce the internal resistance r_0 and the value of the reactive component (L or C) inside dipole $\mathcal{D}$.

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Exercise 7

Two passive linear dipoles D_1 and D_2 connected in series carry the sinusoidal current $i(t) = 5\sqrt{2}\sin(100\pi t + \pi/4)$ A. Across the complete combination and across D_1, the instantaneous voltages are:

$$u(t) = u_1(t) + u_2(t) = 60\sqrt{2}\sin\left(100\pi t + \frac{\pi}{3}\right)\ \text{V}, \qquad u_1(t) = 80\sqrt{2}\sin\left(100\pi t - \frac{\pi}{6}\right)\ \text{V}.$$

1) Give the RMS Fresnel vectors $\vec{U}$ and $\vec{U}_1$ (moduli and polar angles in degrees).

2) Using the scale $20\,\text{V cm}^{-1}$, construct to scale in the Fresnel plane the vector $\vec{U}_2$ representing $u_2(t)$.

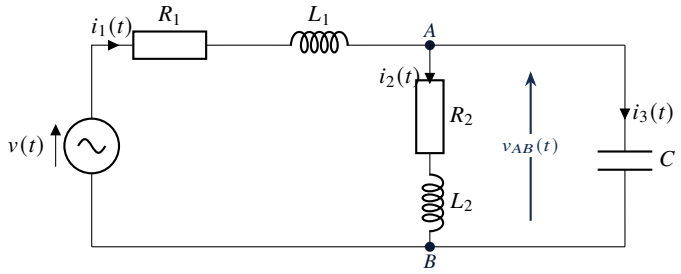
3) Calculate the rectangular projections (U_{2x}, U_{2y}), the RMS voltage U_2, and the initial phase $\theta_2 \in (-180°, +180°]$ of $u_2(t)$. Write the time-domain expression $u_2(t)$.

4) Verify U_2 using the Fresnel law of cosines.

5) Compute the relative phase shifts φ_{u/u_1}, $\varphi_{u_1/i}$, $\varphi_{u_2/i}$, and $\varphi_{u/i}$, and check the additive phase relation linking φ_{u/u_1}, $\varphi_{u_1/i}$, and $\varphi_{u/i}$.

Exercise 8

In the sinusoidal steady-state network shown below, the generator supplies $v(t) = 150\sqrt{2}\sin(1000\,t)$ V ($\omega = 1000\,\text{rad s}^{-1}$, phase origin $\bar{V} = 150\angle 0°$ V). The circuit parameters are $R_1 = 7\,\Omega$, $L_1 = 24\,\text{mH}$, $R_2 = 25\,\Omega$, and $L_2 = 25\,\text{mH}$.



1) Calculate the complex impedances $\bar{Z}_1$ and $\bar{Z}_2$, then determine the complex admittance $\bar{Y}_2 = G_2 + jB_2$ of Branch 2 (R_2, L_2). Compare G_2 with $1/R_2$.

2) For $C = 80\,\mu\text{F}$, calculate the parallel admittance $\bar{Y}_{AB} = \bar{Y}_2 + \bar{Y}_3$, the parallel impedance $\bar{Z}_{AB} = 1/\bar{Y}_{AB}$, and the total input impedance $\bar{Z}_{\text{eq}} = \bar{Z}_1 + \bar{Z}_{AB}$ in both rectangular and polar forms. Deduce the quality factor Q of the complete dipole seen by the source.

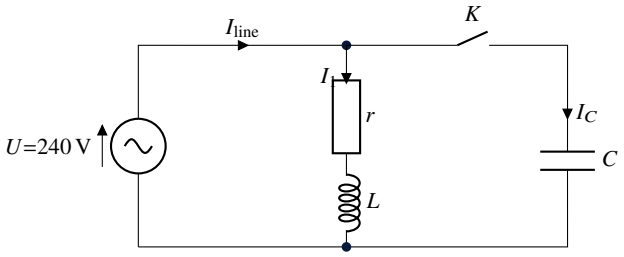
3) Still for $C = 80\,\mu\text{F}$, determine the complex phasors $\bar{I}_1$, $\bar{V}_{AB}$, $\bar{I}_2$, and $\bar{I}_3$, and write the instantaneous source current $i_1(t)$.

4) Verify complex KCL at node A and complex KVL around the outer mesh, and compare the scalar sum of RMS branch currents $I_2 + I_3$ with I_1.

5) Keeping $(R_1, L_1, R_2, L_2, \omega)$ fixed, determine all values of the capacitance C_0 for which the source current $i_1(t)$ is in phase with $v(t)$, and give the corresponding purely resistive input impedance $Z_{\text{eq}}(C_0)$ in each case.

Exercise 9

An industrial coil modeled as an internal resistance r in series with an inductance L is supplied across an AC bus of RMS voltage $U = 240\,\text{V}$ and frequency $f = 50\,\text{Hz}$ (see diagram below). With switch K open, an RMS ammeter and a wattmeter indicate $I_1 = 6.0\,\text{A}$ and active power $P = 864\,\text{W}$.



1) With switch K open:

   (a) Calculate the apparent power S_1, the power factor $\cos\varphi_1$, the signed phase angle φ_1, and the reactive power Q_{r1} absorbed by the coil (specifying the unit of each power quantity). Explain why knowing $\cos\varphi_1$ alone does not fix the sign of φ_1 without knowing the reactive nature of the dipole.

   (b) Determine the coil impedance modulus $|\bar{Z}_{\text{coil}}|$, its internal resistance r, its inductive reactance X_L, and its inductance L.

   (c) Calculate the internal RMS voltages $U_r = r\,I_1$ and $U_L = X_L\,I_1$, and verify the Fresnel voltage relation linking U, U_r, and U_L.

2) Switch K is now closed, placing an ideal capacitor C in parallel across (U, f):

   (a) Express the active power P_C and reactive power $Q_{r,C}$ of the capacitor in terms of U, C, and $\omega = 2\pi f$, and state how closing K affects the total active power drawn from the 240 V bus.

   (b) Determine the capacitance C required to raise the overall power factor of the installation to $\cos\varphi' = 1$.

   (c) Compute the new RMS supply current I'_{line} when $\cos\varphi' = 1$, and determine the percentage reduction in line current achieved by closing K.

Exercise 10

An electrical power network generates sinusoidal AC power, steps up the voltage for high-voltage transmission, and steps it down for low-voltage distribution.

1) A power plant transmits a fixed active power $P = 120\,\text{MW}$ at unity power factor ($\cos\varphi = 1$) along a transmission line whose conductors have resistance R_{line} each.

   (a) Express the Joule power loss P_J per conductor in terms of R_{line} and the line current I, and explain why stepping up the transmission RMS voltage U lowers line losses.

   (b) By what factor do the line Joule losses decrease when the transmission voltage is stepped up from 20 kV to the standard extra-high voltage EHV = 400 kV?

2) After step-down transformation, a low-voltage distribution feeder operates as a 4-wire system with a standard line-to-line RMS voltage $U_{\text{LV}} = 380\,\text{V}$. Identify the four conductors of this distribution line and calculate the corresponding phase-to-neutral RMS voltage V_{LV} supplied to single-phase loads.

3) Consider transmitting the same active power P over distance ℓ at the same RMS voltage U and unity power factor ($\cos\varphi = 1$), using copper conductors of resistivity ρ ($R_{\text{wire}} = \rho\ell/S$):

   - *System 1 (2-wire single-phase line):* 2 conductors of cross-sectional area S_1, each carrying RMS current $I_1 = \dfrac{P}{U}$.
   - *System 3 (3-wire balanced three-phase line):* 3 phase conductors of cross-sectional area S_3 (no neutral wire), each carrying RMS line current $I_3 = \dfrac{P}{\sqrt{3}\,U}$.

   Imposing identical total line Joule losses ($P_{J,1} = P_{J,3}$) in both systems, determine the cross-section ratio S_1/S_3, deduce the ratio M_1/M_3 of total conductor copper mass required by System 1 relative to System 3, and state the fractional extra copper mass needed by single-phase transmission.