



## Direct Integral & Riemann Sums

*Rules: NO Integration by Parts. NO Explicit Variable Changes. Use only algebraic manipulation, trigonometric identities, parity, Limited Developments, and standard primitive tables.*

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### Category 1:

1.  $I_1 = \int_0^{\pi/2} e^x (\cos x - \sin x) dx$

2.  $I_2 = \int_1^{\pi} \frac{x \cos x - \sin x}{x^2} dx$

3.  $I_3 = \int_1^2 \frac{e^x(x-1)}{x^2} dx$

### Category 2:

4.  $I_4 = \int_1^2 \frac{1}{x(1+x^4)} dx$

5.  $I_5 = \int_0^{\ln 3} \frac{1}{e^x + e^{-x}} dx$

6.  $I_6 = \int_0^1 \frac{1}{\sinh x + \cosh x} dx$

### Category 3:

7.  $I_7 = \int_0^1 \frac{x}{x^4 + x^2 + 1} dx$

8.  $I_8 = \int_{-1/2}^0 \frac{1}{x^2 + x + 1} dx$

### Category 4:

9.  $I_9 = \int_{-1}^1 \ln(x + \sqrt{x^2 + 1}) dx$

10.  $I_{10} = \int_{-\pi/3}^{\pi/3} \frac{x^4 \sin(x^3) + x^3 \cos(x)}{1 + x^6} dx$

**Category 5:**

11.  $I_{11} = \int_0^{\pi/4} \tan^2(x) dx$

12.  $I_{12} = \int_{\pi/4}^{\pi/3} \frac{\cos(2x)}{\sin^2(x) \cos^2(x)} dx$

13.  $I_{13} = \int_0^{\pi/2} \sin(3x) \cos(2x) dx$

14.  $I_{14} = \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{1 + \sin(2x)}} dx$

15.  $I_{15} = \int_0^{\pi/2} \frac{1}{1 + \sin x} dx$

16.  $I_{16} = \int_0^{\pi/4} \sin^4 x + \cos^4 x dx$

**Category 6:**

17.  $S_1 = \lim_{n \rightarrow +\infty} \prod_{k=1}^n \exp\left(\frac{k}{n^2 + k^2}\right)$

18.  $S_2 = \lim_{n \rightarrow +\infty} \sum_{k=1}^n \frac{n \arctan(k/n)}{n^2 + k^2}$

19.  $S_3 = \lim_{n \rightarrow +\infty} \sum_{k=1}^n n \left[ 1 - \cos\left(\frac{1}{\sqrt{n^2 + nk}}\right) \right]$

20.  $S_4 = \lim_{n \rightarrow +\infty} \sum_{k=n+1}^{2n} \frac{n}{k^2 - nk + n^2}$